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Power corrections to the differential Drell-Yan cross section

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Abstract: We estimate the power corrections (infrared renormalon contributions) to the coefficient functions for the differential Drell-Yan cross-section $d^2\sigma/dQ^2dy$, where Q^2 is the mass squared and y the rapidity of the produced lepton pair. We employ the dispersive method based on the analysis of one-loop Feynman graphs containing a massive gluon.

Keywords: QCD, NLO Computations, Jets, Hadronic Colliders.

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Contents

1. Introduction

The Drell-Yan process [1] describes the collision of two hadrons and the subsequent production of a lepton pair and a hadronic final state. In the parton model it proceeds simply via the annihilation mechanism where a quark and anti-quark generated by the parent hadrons annihilate to form a photon which decays to a lepton pair. At present day collider energies one is also able to produce the electroweak W and Z bosons on mass shell via this mechanism.

Historically the Drell-Yan process has played an important role in the development of QCD. The parton model became more firmly established when it was realised that it gave a good description of the data in hadron-hadron collisions. In addition it became evident that QCD perturbation theory could be applied to describe strong interaction phenomena when one encountered the very same mass singularities in Drell-Yan calculations as those in the case of deeply inelastic scattering giving rise to the concept of universal functions that control the long-distance dynamics, which one refers to as parton distributions.

Data from hadron-hadron collisions has proved a valuable source from which one has been able to constrain and measure various parton distributions. In particular such data provides a means to extract information on the quark distributions in pions which is inaccessible from DIS experiments [2]. Data on low-mass lepton pair production has been used to study the small x behaviour of parton distributions. For a thorough review of available Drell-Yan data and comparisons with theory the reader is referred to ref. [3].

In this article we choose to concentrate our attention on the cross section $\frac{d^2\sigma}{dQ^2 dy}$ with Q^2 being the mass squared and y the rapidity of the produced lepton pair. Most recently the CDF collaboration have studied the rapidity dependence of the Drell-Yan cross section in a limited rapidity range [4]. In fact from the experimental side there is a wealth of data on various rapidity distributions (see ref. [3]) but progress on the theoretical side is somewhat lacking. While the $\mathcal{O}(\alpha_s)$ perturbative QCD calculations for the above distribution were performed several years ago [5, 6] there is as yet no $\mathcal{O}(\alpha_s^2)$ estimate. Also lacking is any estimate of the non-perturbative power-like corrections which have been extensively studied in many other cases . The aim of this article is to study the power correction to the rapidity distribution which should in principle allow a better description of the data when added to the perturbative predictions. Power correction predictions already exist for the more inclusive cross section $d\sigma/dQ^2$ where a $1/Q^2$ dependence is predicted with a characteristic phase space enhancement [7, 8] .

From a purely theoretical viewpoint the motivation of the work described here is the testing of current ideas on power corrections which seem to indicate that though these contributions are non-perturbative in origin one may suitably extend a perturbative (renormalon) approach to estimate them. For further details on the origin and development of renormalon phenomenology, the reader is referred to refs. [9, 10, 11]. The phenomenological success of such an approach in the case of DIS structure functions [12, 13, 14] is encouraging enough to extend this study to the present case. In view of the relative simplicity of the renormalon calculations that yield predictions for the power corrections and the range and accuracy of current experimental data on several different QCD observables one should be able to undertake a serious and extensive confrontation of these theoretical ideas with the data, a task that has already begun [15]–[19].

This paper is organised as follows. In the next section we give a very brief review of the dispersive treatment of power corrections which has been described in great detail previously [7, 20]. Following this we mention the notation and introduce the kinematical variables relevant to our study. In the following section we describe our results for the power corrections and finally make some concluding remarks.

2. Dispersive approach

The main ideas of the dispersive approach to power corrections can be briefly summed up as below. First one assumes a QED inspired dispersion relation to be formally true in the QCD case so that

$$\alpha_s(k^2) = -\int_0^\infty \frac{d\mu^2}{\mu^2 + k^2}\rho_s(\mu^2)\,, \tag{2.1}$$

with the spectral function

$$\rho_s(\mu^2) = \frac{1}{2\pi i}\left\{\alpha_s(\mu^2 e^{i\pi}) - \alpha_s(\mu^2\, e^{-i\pi})\right\}\,. \tag{2.2}$$

Thus one is assuming that the QCD coupling is well behaved in the infra-red and the only singularity is a discontinuity on the negative real axis of its argument.

Non-perturbative effects at long distances are expected to give rise to a modification to the perturbatively-calculated strong coupling at low scales, $\delta\alpha_s(\mu^2) = \alpha_s(\mu^2) - \alpha_s^{\rm PT}(\mu^2)$, $\alpha_s^{\rm PT}(\mu^2)$ being the perturbatively-calculated running coupling [7]. The spectral function (2.2) receives the corresponding modification

$$\delta\rho_s(\mu^2) = \frac{1}{2\pi i}\mathrm{Disc}\left\{\delta\alpha_s(-\mu^2)\right\}\,. \tag{2.3}$$

To consider the effect of the above on an observable F one assumes the implicit inclusion of a gauge invariant set of higher order graphs (which in a large N_f approximation just reduce to quark bubble insertions) combined with single gluon emission has the effect of generating the running coupling in the one-loop calculation of F [20]. In practice the running coupling can only be reconstructed in this manner, for a sufficiently inclusive observable like the one we study in this article. The above considerations allow one to write [7]

$$F = \int_0^\infty \frac{d\mu^2}{\mu^2}\rho_s(\mu^2)\left[\mathcal{F}\left(\frac{\mu^2}{Q^2}\right) - \mathcal{F}(0)\right], \tag{2.4}$$

where $\mathcal{F}$ is the one loop correction to the observable, computed with a finite gluon mass μ^2. Then the non-perturbative region contributes to F through eq. (2.3) and one finds

$$\delta F = \int_0^\infty \frac{d\mu^2}{\mu^2}\delta\alpha_s(\mu^2)\mathcal{G}\left(\frac{\mu^2}{Q^2}\right), \tag{2.5}$$

where setting $\mu^2/Q^2 = \epsilon$

$$\mathcal{G}(\epsilon) = -\frac{1}{2\pi i}\mathrm{Disc}\left\{\mathcal{F}(-\epsilon)\right\}\,. \tag{2.6}$$

Since $\delta\alpha_s(\mu^2)$ is limited to low values of μ^2 the asymptotic behaviour of δF at high Q^2 is given by its behaviour in the limit $\epsilon \to 0$. Clearly only terms that are non-analytic in ϵ in the small ϵ behaviour of $\mathcal{F}$, yield non-perturbative modifications to F within the above approach. In the present case the characteristic function $\mathcal{F}$ will be found to have the general small ϵ behaviour

$$\mathcal{F} \sim \frac{C_F}{2\pi}\left(C_1\left\{x\right\}\epsilon\ln^2\epsilon + C_2\left\{x\right\}\epsilon\ln\epsilon\right), \tag{2.7}$$

with $\{x\}$ denoting the phase space dependence that we compute here. According to (2.5) and (2.6) the above ϵ dependence translates into the power correction

$$\frac{\mathcal{A}_2}{Q^2}\left[C_2\left\{x\right\} + 2C_1\left\{x\right\}\ln\left(\frac{\mathcal{B}_2}{Q^2}\right)\right], \tag{2.8}$$

with the parameters $\mathcal{A}_2$ and $\mathcal{B}_2$ being

$$\begin{aligned} \mathcal{A}_2 &= \frac{C_F}{2\pi} \int_0^\infty \frac{d\mu^2}{\mu^2}\, \mu^2\, \delta\alpha_s(\mu^2) \\ \ln \mathcal{B}_2 &= \frac{1}{\mathcal{A}_2} \frac{C_F}{2\pi} \int_0^\infty \frac{d\mu^2}{\mu^2}\, \mu^2\, \ln \mu^2\, \delta\alpha_s(\mu^2)\,. \end{aligned} \tag{2.9}$$

Then one invokes the universality assumption, in that if one believes that the concept of α_s can be meaningfully extended to small scales, it can be done in an observable and indeed process independent fashion. This would allow us to extract the value of the above defined moments of the coupling, $\mathcal{A}_2$ and $\mathcal{B}_2$, from any experimental data where the same power correction is obtained and use it to fit Drell-Yan data. Recent studies of $1/Q$ corrections to event shape variables in e^+e^- annihilation [18] and in DIS [19] lend support to this idea . Studies of the $1/Q^2$ non-singlet contribution to DIS structure functions suggest that $\mathcal{A}_2 \simeq 0.2\,\mathrm{GeV}^2$ [12]. We do not know the value of the parameter $\mathcal{B}_2$ from any experimental data and hence it can be treated as a free parameter in the fit to the Drell-Yan case.

On the other hand as has been explained in detail in previous articles (see for instance ref. [21]) the parameter $\mathcal{A}_2$ and indeed any other moment of the modified coupling, $\delta\alpha_s$, depends on the order of the perturbative result with which one is interested in merging the non-perturbative piece. The above DIS value is relevant to merging with the next-to-leading order (NLO) QCD prediction while in the present case of the differential Drell-Yan cross-section one only knows the leading QCD correction to the parton model. However we use the DIS value for illustrative purposes here and in the expectation that the NLO result will be available soon.

3. Definitions and kinematics

We wish to compute the power corrections to the observable $d^2\sigma/dQ^2dy$ where Q^2 is the mass squared and y is the rapidity of the produced lepton pair. To lowest order the expression for the above quantity is simply

$$\frac{d^2\sigma}{dQ^2dy} = \frac{\sigma_0}{Ns}\left[\sum_q e_q^2 \left\{ f_{q/A}(z_1,Q^2) f_{\bar{q}/B}(z_2,Q^2) + (q \leftrightarrow \bar{q})\right\}\right], \tag{3.1}$$

where

$$\sigma_0 = \frac{4\pi\alpha^2}{3Q^2}\,,$$

N is the number of colours and s the center-of-mass energy squared. In addition e_q refers to the charge of flavour q in the flavour sum above and z_1, z_2 refer to the momentum fractions carried by the quark and antiquark, of the parent hadrons A and B while the f functions are the corresponding parton distributions. We explicitly have, in the hadronic centre of mass frame, the parton momenta (neglecting their small intrinsic k_t)

$$\begin{aligned} p_1 &= \frac{\sqrt{s}}{2}(z_1, 0, 0, z_1)\,, \\ p_2 &= \frac{\sqrt{s}}{2}(z_2, 0, 0, -z_2)\,. \end{aligned} \tag{3.2}$$

The invariant mass of the produced lepton pair is then simply given by

$$Q^2 = (p_1 + p_2)^2 = s z_1 z_2\,, \tag{3.3}$$

while its rapidity is

$$y = \frac{1}{2}\ln\left(\frac{E + P_L}{E - P_L}\right) = \frac{1}{2}\ln\left(\frac{z_1}{z_2}\right) \tag{3.4}$$

which yield

$$z_1 = \sqrt{\tau}e^{y}\,, \qquad z_2 = \sqrt{\tau}e^{-y}\,, \qquad \tau = \frac{Q^2}{s}\,. \tag{3.5}$$

Beyond the naive parton model one has to consider radiative corrections to the above picture, wherein initial partons carrying momentum fractions z_1 and z_2 degrade their momenta via gluon radiation before annihilating to form the lepton pair. Hence the simple correspondence (3.5) between the momentum fractions of the initiating partons and the mass and rapidity of the lepton pair no longer holds beyond lowest order. We define for later use kinematic variables x_1 and x_2 such that

$$x_1 = \sqrt{\tau}e^{y}\,, \qquad x_2 = \sqrt{\tau}e^{-y} \tag{3.6}$$

and which at Born level are just equal to the initial parton momentum fractions.

For computing the renormalon contribution to the desired observable we shall need the squared matrix element for the QCD radiative process[1]

$$q(p_1) + \bar{q}(p_2) \to \gamma^*(q) + g(k)\,, \tag{3.7}$$

which takes the simple form

$$\mathcal{M}_{DY} = \frac{\hat{s}^2 + Q^4(1+\epsilon)^2}{\hat{u}\hat{t}} - 2 - \epsilon Q^4\left(\frac{1}{\hat{u}^2} + \frac{1}{\hat{t}^2}\right) \tag{3.8}$$

where $\hat{s}, \hat{t}, \hat{u}$ are Mandelstam invariants defined as

$$\hat{s} = (p_1 + p_2)^2\,, \qquad \hat{t} = (p_1 - k)^2\,, \qquad \hat{u} = (p_1 - q)^2\,, \tag{3.9}$$

[1]For computing the power correction we are only considering the annihilation contribution. In general there is also the QCD Compton scattering process to consider, beyond the naive parton model. We shall comment further on this in the final section.

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and which satisfy the relation

$$\hat{s}+\hat{t}+\hat{u}=Q^2+\mu^2\,, \tag{3.10}$$

with $\mu^2=\epsilon Q^2$ being the squared gluon mass and $\hat{s}=z_1z_2s$.

The rapidity y can readily be obtained in terms of the Mandelstam invariants and the momentum fractions z_1 and z_2 as

$$y=\frac{1}{2}\ln\left(\frac{Q^2-\hat{t}}{\hat{s}+\hat{t}-\mu^2}\,\frac{z_1}{z_2}\right), \tag{3.11}$$

which is equivalent to

$$\begin{aligned}\hat{t}&=\frac{Q^2x_2z_1-x_1z_2(\hat{s}-\mu^2)}{x_2z_1+x_1z_2}\,,\\ \hat{u}&=\frac{Q^2x_1z_2-x_2z_1(\hat{s}-\mu^2)}{x_2z_1+x_1z_2}\,,\end{aligned} \tag{3.12}$$

with x_1 and x_2 defined as before. The phase space for massive gluon emission is given by

$$(z_1-x_1)(z_2-x_2)\geq\frac{\mu^2}{s}\,. \tag{3.13}$$

Putting the above together we obtain the required differential cross section

$$\frac{d^2\hat{\sigma}}{dQ^2dy}=A\frac{\tau(\tau+z_1z_2-\epsilon\tau)}{(z_1z_2)(z_1x_2+z_2x_1)^2}\,\mathcal{M}(s,x_1,x_2,z_1,z_2,\epsilon) \tag{3.14}$$

where $\mathcal{M}$ is just the matrix element (3.8) with the substitutions of eq. (3.12) and $A=16\alpha^2\alpha_s e_q^2/27Q^2s$. The hadron level result is related to the corresponding partonic quantity by folding with parton distribution functions as below

$$\frac{d^2\sigma}{dQ^2dy}=\sum_q\int_{x_1}^1\int_{x_2}^1 dz_1dz_2\frac{d^2\hat{\sigma}}{dQ^2dy}\Theta\left((z_1-x_1)(z_2-x_2)-\frac{\mu^2}{s}\right)\mathcal{F}_q(z_1,z_2)\,, \tag{3.15}$$

where for brevity we defined

$$\mathcal{F}_q(z_1,z_2)=\left\{f_{q/A}(z_1,Q^2)f_{\bar{q}/B}(z_2,Q^2)+(q\leftrightarrow\bar{q})\right\}. \tag{3.16}$$

Next we introduce the variables $\xi=x_1/z_1$ and $\zeta=x_2/z_2$ in terms of which eq. (3.15) assumes the familiar form

$$\frac{d^2\sigma}{dQ^2dy}=\sum_q e_q^2\frac{C_F\alpha_s}{2\pi}\int_{x_1}^1\int_{x_2}^1\frac{d\xi}{\xi}\frac{d\zeta}{\zeta}C(\xi,\zeta,\epsilon)\mathcal{F}\left(\frac{x_1}{\xi},\frac{x_2}{\zeta}\right)\Theta((1-\xi)(1-\zeta)-\epsilon\xi\zeta) \tag{3.17}$$

in writing which we have dropped an uninteresting overall constant factor σ_0/Ns, which also appears in the Born cross section to which we shall normalise the result subsequently. The coefficient function $C(\xi,\zeta,\epsilon)$ then takes the form

$$C(\xi,\zeta,\epsilon) = \frac{2(1+\xi\zeta-\epsilon\xi\zeta)(1+\xi^2\zeta^2(1+\epsilon)^2)}{(1-\xi^2-\epsilon\xi\zeta)(1-\zeta^2-\epsilon\xi\zeta)} - 4\frac{\xi\zeta(1+\xi\zeta-\epsilon\xi\zeta)}{(\xi+\zeta)^2} - \\ - 2\epsilon\xi\zeta(1+\xi\zeta-\epsilon\xi\zeta)\left[\frac{\xi^2}{(\xi^2-1+\epsilon\xi\zeta)^2}+\frac{\zeta^2}{(\zeta^2-1+\epsilon\xi\zeta)^2}\right]. \quad (3.18)$$

Next taking moments of the convolution equation (3.17) we find as usual (note the different powers)

$$\int_0^1\int_0^1 dx_1\,dx_2\,x_1^N\,x_2^M\frac{d^2\sigma}{dQ^2dy} = \sum_q e_q^2\frac{C_F\alpha_s}{2\pi}\,\tilde{C}(N,M,\epsilon)\,\tilde{\mathcal{F}}_q(N,M) \quad (3.19)$$

where

$$\begin{aligned}\tilde{C}(N,M,\epsilon) &= \int_0^1\int_0^1 \xi^N\zeta^M\,C(\xi,\zeta,\epsilon)\,\Theta((1-\xi)(1-\zeta)-\epsilon\xi\zeta)d\xi\,d\zeta\\ \tilde{\mathcal{F}}_q(N,M) &= \tilde{f}_{q/A}(N)\,\tilde{f}_{\bar{q}/B}(M)+(q\leftrightarrow\bar{q})\end{aligned} \quad (3.20)$$

and the $\tilde{f}$ functions represent the Mellin transforms of the quark and anti-quark density functions.

As explained previously, to extract the power corrections we have to look for non-analytic behaviour in ϵ, in the small ϵ expansion of the Mellin transformed coefficient functions $\tilde{C}(N,M,\epsilon)$. This non-analyticity will manifest itself (in the present case) through the appearance of a logarithmic dependence on ϵ in addition to the usual logarithmic divergences generated by the soft and collinear regions of integration. These divergences are of course cancelled by virtual corrections in the case of the infrared and by absorbing the collinear divergences into the definition of the parton densities. We are then left with terms like $\epsilon\ln^2\epsilon$ and $\epsilon\ln\epsilon$ which, as shown in ref. [20], are in one to one correspondence with infrared renormalon poles in the Borel plane and will generate $1/Q^2$ power corrections. We neglect similar terms that appear at order ϵ^2 and higher orders in ϵ since they will induce sub-leading $\mathcal{O}(1/Q^4)$ power corrections in which we are not phenomenologically interested.

4. Power corrections

In taking the Mellin transforms of the coefficient functions as mentioned in (3.19) one finds that the second and third pieces of the expression on the right-hand-side of eq. (3.18) do not produce any logarithmic divergences but contribute only to the power corrections through the appearance of an $\epsilon\ln\epsilon$ term. The collinear and infrared divergences lie in the first piece on the right-hand-side of (3.18). The Mellin transforms of this piece are cumbersome to evaluate directly and the result is most easily arrived at after a further change of integration variables:

$$\hat{\tau} = \xi\zeta\,, \qquad \eta = \frac{\xi}{\zeta}\,. \quad (4.1)$$

In terms of these variables the double Mellin transform of the above mentioned piece takes the simple form

$$\int_0^{1/(1+\sqrt{\epsilon})^2} d\hat{\tau} \int_{\eta_1}^{\eta_2} d\eta\, \hat{\tau}^{(N+M)/2}\, \eta^{(N-M)/2}\, \frac{(1+\hat{\tau}-\epsilon\hat{\tau})(1+\hat{\tau}^2(1+\epsilon)^2)}{(1-\eta\hat{\tau}-\epsilon\hat{\tau})(\eta-\hat{\tau}-\epsilon\eta\hat{\tau})}\,. \tag{4.2}$$

In the above equation the limits of the η integration are given by

$$\eta_1 = \frac{1}{\eta_2} = \frac{1}{2\hat{\tau}}(\Lambda + 2\hat{\tau} + \sqrt{\Lambda^2 + 4\Lambda\hat{\tau}})\,, \qquad \Lambda = 1 - 2\hat{\tau} + \hat{\tau}^2 - 2\hat{\tau}\epsilon - 2\hat{\tau}^2\epsilon + \epsilon^2\hat{\tau}^2. \tag{4.3}$$

Taking the η moments is now an easy task but inserting the limits (4.3) complicates the extraction of the τ moments. In particular one has to evaluate the form (apart from other relatively straightforward integrals)

$$\int_0^{1/(1+\sqrt{\epsilon})^2} d\hat{\tau}\, \hat{\tau}^{\omega} \left(\frac{1+\hat{\tau}^2+\hat{\tau}^2\epsilon^2+2\epsilon\hat{\tau}^2}{1-\hat{\tau}-\epsilon\hat{\tau}}\right) \tanh^{-1}\left[\frac{\sqrt{\Lambda}}{1-\hat{\tau}-\epsilon\hat{\tau}}\right] \tag{4.4}$$

where we use ω to denote a generic power which depends on M and N. The above form has been evaluated in ref. [7] and we shall not describe the manipulations that lead to the evaluation of integrals of the above type but refer the reader to ref. [7, section 4.6 and appendix A] for the details. The moments of the second and third pieces on the right hand side of eq. (3.18) are most easily evaluated directly in ξ, ζ space.

Putting together the contribution of all the pieces and including the time-like virtual corrections (which are independent of M and N and were computed in ref. [7]) we obtain the result for the total contribution as follows:

$$\tilde{C}^{\mathrm{R+V}}(N,M,\epsilon) = \tilde{C}_0 \ln\epsilon + \tilde{C}_1\, \epsilon \ln^2\epsilon + \tilde{C}_2 \epsilon \ln\epsilon\,, \tag{4.5}$$

where

$$\tilde{C}_0 = \left[2S_1(N+1) + 2S_1(M+1) - 3 + \frac{1}{M+1} + \frac{1}{N+1} + \frac{1}{N+2} + \frac{1}{M+2}\right] \tag{4.6}$$

$$\tilde{C}_1 = -\left[\frac{N}{2} + \frac{M}{2} + 3\right] \tag{4.7}$$

$$\begin{aligned}\tilde{C}_2 = \Bigg[&-(M+N+2)S_1(N+1) - (M+N+2)S_1(M+1) + 2MN + \\ &+ 5M + 5N + 4 + \frac{N-M}{N+1} + \frac{M-N}{M+1} + \frac{N-2M}{2(N+2)} + \frac{M-2N}{2(M+2)} + \\ &+ \frac{N-2M-3}{2(N+3)} + \frac{M-2N-3}{2(M+3)}\Bigg]\end{aligned} \tag{4.8}$$

and

$$S_1(N) = \sum_{j=1}^{N-1} \frac{1}{j} = \psi(N) + \gamma_E\,. \tag{4.9}$$

Notice the absence of a $\ln^2\epsilon$ term which cancels in the sum of the real and virtual pieces. In addition, the logarithmic divergence in the cut-off (gluon mass) is absorbed into the structure functions and will not concern us any more. We shall instead concentrate our attention on the corrections generated by the $\epsilon\ln^2\epsilon$ and $\epsilon\ln\epsilon$ piece of eq. (4.5). As a check on our result above, we note that in the diagonal limit $M = N$ we recover the result obtained in ref. [7] provided one changes N to $N-1$ in accordance with the different definition of the moments adopted in that reference. This agreement is expected as in the special case, $M = N$, our calculation reduces to just extracting the τ dependence of the power corrections. The above results can be easily expressed in ξ, ζ space and the convolutions with the parton densities performed as prescribed in eq. (3.17).

5. Discussion

For the purposes of illustration we assume that $f_{q/A} = f_{\bar{q}/B} = q$ with the $q \leftrightarrow \bar{q}$ term being similarly labelled $\bar{q}$. In reality of course such assumptions about the density functions will depend on the beam and the target used in the relevant experiment but our predictions can be easily adjusted to every case.

Then we find that performing the convolution and extracting the lowest (Born) order result allows one to write :

$$\frac{d^2\sigma}{dQ^2dy} = \left(\frac{d^2\sigma}{dQ^2dy}\right)_B \left[1 + \frac{\mathcal{A}_2}{Q^2} H(x_1, x_2)\right], \tag{5.1}$$

where the power correction function $H(x_1, x_2)$ can be expressed as

$$\begin{aligned} H(x_1,x_2) = \frac{1}{f(x_1,x_2)} \sum_q e_q^2 \Bigg[&\{x_1 q'(x_1) q(x_2) + x_2 q'(x_2) q(x_1) - 4q(x_1)q(x_2)\} \ln\left(\frac{\mathcal{B}_2}{Q^2}\right) + \\ &+ 2x_1x_2\, q'(x_1)q'(x_2) - 3x_1\, q'(x_1)q(x_2) - \\ &- 3x_2\, q'(x_2)q(x_1) - q(x_1)\, I(x_2) - q(x_2)\, I(x_1) - \\ &- x_1\, q'(x_1)\, K(x_2) - x_2\, q'(x_2)\, K(x_1) + q \leftrightarrow \bar{q} \Bigg] \end{aligned} \tag{5.2}$$

with the following definitions:

$$f(x_1,x_2) = \sum_q e_q^2 (q(x_1)q(x_2) + q \leftrightarrow \bar{q}) \tag{5.3}$$

$$I(z) = \int_z^1 \frac{dy}{y} \left[\frac{1}{(1-y)_+} \left(\frac{z}{y} q'\left(\frac{z}{y}\right)\right) + 2y^2\, q\left(\frac{z}{y}\right)\right] \tag{5.4}$$

$$K(z) = \int_z^1 \frac{dy}{y} \left[\frac{1}{(1-y)_+} - (1 + y + y^2)\right] q\left(\frac{z}{y}\right). \tag{5.5}$$

In the above formulae the '+' distributions have their usual meaning, the prime symbol on the q denotes a derivative and the scale (Q^2) dependence of the parton

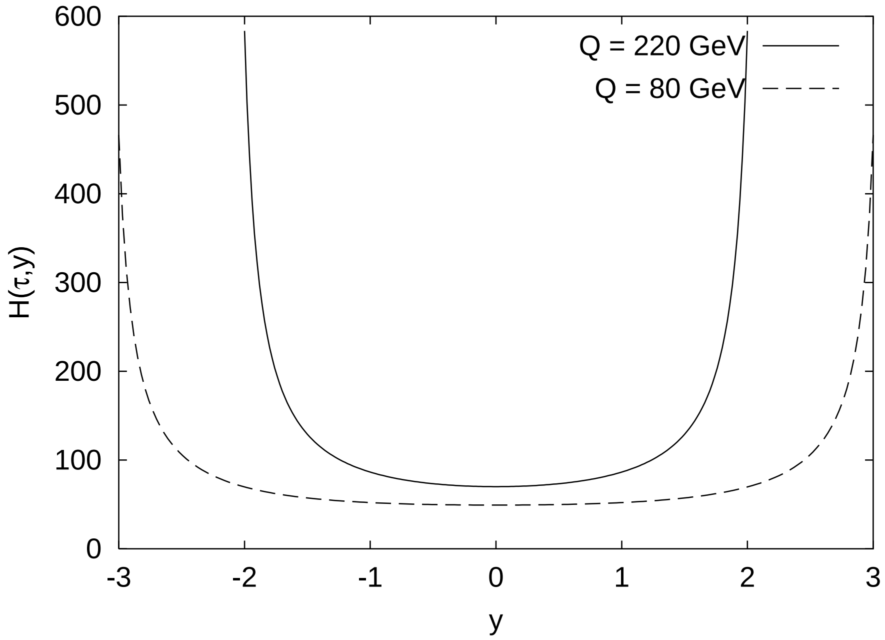

Figure 1: Plot of the rapidity dependence of the power correction function $H(\tau, y)$. The solid line corresponds to $\tau = 0.0149$, $|y| < 2$ while the dashed line corresponds to $\tau = 0.0019$, $|y| < 3$.

distributions is understood. Note that the symmetry under the exchange of x_1 and x_2 implies that the power correction is a symmetric function about $y = 0$ with our assumption about the parton densities, which would be valid in the $p\bar{p}$ case. For a general beam and target, asymmetry will be induced purely by the differing parton densities in the beam and target.

A plot of the function H against the rapidity y is shown in figure 1, for two different Q values which correspond to two different values of τ. The value of $\sqrt{s}$ was chosen to be 1.8 TeV, and the plots were made using the MRSA valence parton distributions [22]. The value of the parameter $\mathcal{B}_2$ was chosen to be $1\,\mathrm{GeV}^2$ for illustrative purposes; in principle there is no reason why it should have a value close to the one chosen here.

Both curves shown reflect a similar behaviour, namely that the power correction is quite flat until one starts to reach the edge of the rapidity range shown in either case. For example, in the plot at 80 GeV as one gets closer to $y = 3$, we start approaching the region where $x_1 = \sqrt{\tau}e^y$ is near unity. In fact for $y = 2.9$ and $Q = 80\,\mathrm{GeV}$ one finds $x_1 = 0.80$. Similarly as one progresses towards more negative rapidity values, $x_2 = \sqrt{\tau}e^{-y}$ starts approaching unity. When either x_1 or x_2 gets close to unity an explosive behaviour of the power correction is witnessed. This behaviour is a reflection of the singular nature (as $\xi, \zeta \to 1$) of the derivatives of delta functions, which are obtained on inverse transforming the Mellin-space coefficient functions $\tilde{C}_1$ and $\tilde{C}_2$.

At the edge of the rapidity range for the 80 GeV plot, the overall effect of the power correction $\mathcal{A}_2 H(\tau, y)/Q^2$ is large enough to be comparable to $\alpha_s^2(Q)$ which makes it an important effect to consider while comparing perturbative predictions with the data. For larger Q values, although there is a logarithmic enhancement of the power correction in this case, the suppression by Q^2 should reduce the significance of the correction.

In the more inclusive case of the cross-section differential in Q^2 the explosion of the power correction will only be important as *both* x_1 and x_2 approach unity, in other words in the limit $\tau \to 1$ [7]. This region is probably beyond any experimental interest as the Q value is too high (close to the centre of mass energy) in that case. Hence while the power correction should not be an important consideration in the Q^2 distribution, its presence should be felt in the combined Q^2, y distribution, especially at moderate Q values and towards the edge of the allowed rapidity range.

Lastly we comment on the fact that at higher rapidities one would expect the QCD Compton scattering process to become significant. We have not taken this into account here as it is not yet completely clear how to treat renormalon contributions from incoming gluons as a genuinely gluonic contribution. The only treatment suggested till now is to compute the renormalon contribution, treating the gluon as an internal line, radiated off a quark leg which necessitates an $\mathcal{O}(\alpha_s^2)$ treatment similar to that provided in [23] for e^+e^- annihilation and in [24, 25, 26] for DIS. This procedure can in principle be applied here in the Drell-Yan case. However it leaves the question of how one may unambiguously factor off the quark to gluon splitting in order to be able to convolute with the gluon density, which rapidly grows at small momentum fractions x. Till this issue is better understood we postpone further discussion on this topic.

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